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Bianchi Type-I Cosmological Model with Modified Chaplygin Gas in Einstein's Theory of Gravitation

Avinash R. Nagpure

Department of Mathematics, Jawaharlal Nehru Arts, Commerce and Science College Wadi, Nagpur-440023 (India)

Abstract – *Evolution of a homogeneous, anisotropic universe given by a Bianchi type- I cosmological model with modified Chaplygin gas has been considered. It is assumed that the equation of state of this modified model is valid from the radiation era to the Λ cosmological model. We have applied state-finder parameters in characterizing different phase of the model.*

I-INTRODUCTION

An accelerated expansion of the universe and lead to the search for a new type of matter which violated the strong energy condition i.e. p+3p<0 is satisfied. And referred in the observations of the luminosity of type-I supernovae indicate [1,2]. The matter content such a condition to be satisfied at a certain stage of evolution of the universe. The dark energy represented by a scalar field is called quintessence the transition from a universe filled with matter to an exponentially expanding universe does not necessarily require the presence of a scalar field as the only alternative. In particular, one can try another alternative by using an exotic type of fluid—the so called Chaplygin gas which obeys an equation of state such as $p = -B/p$[3,4], where p and p are respectively the pressure and energy density and B is positive constant. The above equation was modified to the form $p = -\frac{B}{p^{a}{with}}\, 0 <\propto\leq 1$, this model gives the cosmological evolution from initial dustlike matter to an asymptotic cosmological constant and a fluid obeying an equation of state p=$\propto p$. This generalized model has been studied previously [5-7].

The state finder diagnostic a observations may be used to discriminate between different dark energy models. The statefinder diagnostic pair is constructed from the scale factor a(t) and its up to the third order derivatives as follows

$$r = \frac{\dot{a}}{aH^3} \text{ and } s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \qquad (1)$$

Where H and q ($=\frac{a\ddot{a}}{a^3}$) are the Hubble parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The parameter r forms the next step in the hierarchy of geometrical consmological parameters after H and q.
In work, we consider a more general modified Chaplygin gas obeying an equation of state [9]

$$\rho = Ap - \frac{B}{\rho^{\propto}} \text{ with } 0 \leq\propto\leq 1. \qquad (2)$$

This equation of state shows a radiation era (when A =1/3) at one extreme (when $a(t)$ is vanishing small) and a model at the other extreme (when the $a(t)$ is infinitely large). In all stages it's shows a mixture. Also in between there is one stage when the pressure vanishes and the matter content is equivalent to pure dust. We have the effective Lagrangian.

$$L_{\emptyset} = \frac{1}{2}\emptyset^2 - V(\emptyset). \qquad (3)$$

Where ∅ scalar field and v(∅) potential.
In the paper of Gorini et al. [5,6] it has been shown that the simple flat Friedmann model with Chaplygin gas can equivalently be described in terms of a homogeneous minimally coupled scalar field ∅. In this case FRW equations for Chaplygin gas fit into Barrow's scheme

[10]. Folowing Barrow [11], Kamenshchik et al. [3,4,12] have obtained homogeneous scalar field $\emptyset(t)$ and a potential $V(\emptyset)$ to describe Chaplygin cosmology. Debnath et al. [13] have studied the role of modified Chaplygin gas in accelerated universe. In the work we have studied the influence of modified Chaplygin gas in Bianchi type-I universe.

II- Modified Chaplygin Gas in Bianchi Type-I Universe

In the Bianchi type-I universe the metric of a homogeneous and anisotropic universe is.

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \quad (4)$$

Where A, B, C are functions of t only.

The Einstein field equations for the metric (4) are written in form

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = p. \quad (5)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p. \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p. \quad (7)$$

$$\frac{\ddot{A}}{B} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p. \quad (8)$$

Where ρ and ρ are the energy density and pressure, respectively (choosing 8πG=c=1). We define

V=ABC (9)

Subtracting (6) from (7) [14-16]. We get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \quad (10)$$

Using (9) in (10), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \quad (11)$$

Integrating the above equation, we get

$$\frac{A}{B} = d_1 exp\ \left(x_1 \int \frac{dt}{V}\right), d_1 = \text{constant}, x_1 = \text{constant}. \quad (12)$$

By subtracting (8) from (6) and (7) from (8), we can obtain similarly

$$\frac{A}{C} = d_2 exp\left(X_2 \int \frac{dt}{V}\right), \quad (13)$$

$$\frac{B}{C} = d_3 exp\left(X_3 \int \frac{dt}{V}\right) \quad (14)$$

Where d_2, d_3, x_2, x_3 are integration constants.

In view of (9), we find the following relation between the constant s$d_1, d_2, d_3, x_{1,} x_2, x_3$

$$d_2 = d_1 d_3, \qquad x_2 = x_1 + x_3$$

Finally from (12), (13) and (14), we write $A(t), B(t), and\ C(t)$ in the explicit form.

$$A(t) = D_1 V^{1/3} exp\left(X_1 \int \frac{dt}{V(t)}\right), \quad (15)$$

$$B(t) = D_2 V^{1/3} exp\left(X_2 \int \frac{dt}{V(t)}\right), \quad (16)$$

$$C(t) = D_3 V^{1/3} exp\left(X_3 \int \frac{dt}{V(t)}\right), \quad (17)$$

Where D_1(i= 1,2,3) and X_1(i=1,2,3) satisfy the relation $D_1 D_2 D_3$=1 and $X_1 + X_2 + X_3 = 0$.

Now, adding (6),(7),(8) and three times (5), we get

$$\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) + 2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right) = \frac{3k}{2}(\text{p-p}). \quad (18)$$

From (9) we have

$$\frac{\ddot{V}}{V}\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) + 2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right) \quad (19)$$

From (18) and (19) we obtain

$$\frac{\ddot{V}}{V} = \frac{3}{2}(\rho - p) \quad (20)$$

The energy conservation equation is

$$\dot{p} + \frac{\dot{V}}{V}(\rho + p) = 0. \quad (21)$$

From (2) and (21), we have

$$\rho = \left[\frac{B}{1+A} = +\frac{C}{V^{[1+A][1+\vartheta]}}\right]^{\frac{1}{1+\vartheta}} \quad (22)$$

where c is an arbitrary integration constant.

Now for a small value of the scale factors A(t) and B (t) we have

$$\rho \cong \frac{C^{\frac{1}{1+\vartheta}}}{V^{\frac{1}{1+A}}} \quad (23)$$

which is very large and corresponds to the universe dominated by an equation of state

$\rho \cong$ Ap. (24)

Then from (20), we have

$$\frac{\ddot{V}}{V} = \frac{3}{2}(1 - A)\frac{C^{\frac{1}{1+\vartheta}}}{V^{\frac{1}{1+A}}} \quad (25)$$

Which gives

$$\int \frac{dV}{\sqrt{3C^{\frac{1}{1+\vartheta}}v^{(1-A)} + C_1}} = t = t_0 \quad (26)$$

where C_1 = integration constant

For C_1= 0,(25) gives

V=$\left[\frac{3(A+1)}{4}\right]^{\frac{1}{A+3}} C^{\frac{1}{(1+\sigma)(1+A)}} t^{\frac{2}{A+1}}$ (27)

From (15), (16), (17) and (26), we get

A(t)=$D_1 L$ exp [X_1M] (28)

B(t)= $D_2 L$ exp [X_2M] (29)

C(t)= $D_3 L$ exp [X_3M] (30)

WhereL=$\left[\frac{3(A+1)}{4}\right]^{\frac{1}{3(A+1)}} C^{\frac{1}{3(1+\alpha)(1+A)}}$

$t^{\frac{2}{3(1+A)}}$,M=$\left[\frac{4}{3(A+1)C^{1/1+\alpha}}\right]^{\frac{1}{(A+1)}} \frac{(A+1)}{(A-1)} t^{\frac{(A-1)}{(A+1)}}$

From (23) and (26) we get

ρ=$\frac{4}{3(A+1)^2}\frac{1}{t^2}$ (31)

and p= $\frac{4A}{3(A+1)^2}\frac{1}{t^2}$ (32)

The expansion scalar θ, the mean anisotropy parameter A, the shear scalar a^2 and the deceleration parameter q. they are defined as [17]

$\theta = 3H$

A =$\frac{1}{3}\sum_{i-1}^{3}\left(\frac{\Delta H_1}{H}\right)^2$ (33)

A= $\sigma^2 = \frac{1}{2}(\sum_{i-1}^{3} H_i^2 - 3H^2) - \frac{3}{2}AH^2$ (34)

q=$\frac{d}{dt}\left(\frac{1}{H}\right) - 1$ (35)

From (32)-(35), we obtain

θ=$\left(\frac{2}{A-1}\right)\frac{1}{t}$ (36)

A=$\frac{3X^2(A+1)^2}{4}\left\{\frac{4}{3(A+1)C^{\frac{1}{1+\alpha}}}\right\}^{\frac{1}{1+A}} t^{2\left(\frac{A-1}{A+1}\right)}$ (37)

σ^2= $\frac{X^2}{2}\left\{\frac{4}{3(A+1)C^{\frac{1}{1+\alpha}}}\right\}^{\frac{1}{1+A}} t^{-\left(\frac{4}{A+1}\right)}$ (38)

q=$\frac{1}{2}(1 + 3A)$ (39)

where $X^2 \equiv X_{1^2} + X_{2^2} + X_{3^2}$

From (1) and (26)

r= $\frac{9}{2}A(A+1) + 1$ (40)

and

s=2(A+1) (41)

For a large value of the scale factors $A(t), B(t), C(t)$, we obtain

$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ (42)

and

$\rho \approx -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ (43)

Then from (20), we have

$\frac{\ddot{V}}{V} = \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ (44)

Which gives

V=$\sqrt{\frac{2C_2}{3}}\left(\frac{1+A}{B}\right)^{\frac{1}{2(1+\alpha)}}$ sinh $\left\{3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} t + C_2\right\}$ (45)

Where C_1 and C_2 are integration constants.

From (15), (16), (17) and (45) we get

A(t)=D_1N exp [$-X_1 Q$] (46)

B(t)=D_2N exp [$-x_2 Q$] (47)

C(t)=$D_3 N$ exp [$-x_3 Q$] (48)

Where

N= , $Q = \left[-\frac{1}{K_1 K_2} cot^{-1}[cosh(K_2 t + C_2)]\right]$,

$K_1 \equiv \sqrt{\frac{2C_1}{3}}\left(\frac{1+A}{B}\right)^{\frac{1}{2(1+\alpha)}}$ and $K_2 \equiv 3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ are constants.

The physical quantities of observational interest in cosmology are

θ=K_2 $coth$ $(K_2 t = C_2)$, (49)

A=$\frac{3X^2}{K_{2^4}}\frac{sinh^4(K_2t+C_2)}{cosh^2(K_2t+C_2)[1+cosh^2 sinh^4(K_2t+C_2)^2]}$ (50)

$\sigma^2 = \frac{X^2}{2}\frac{sinh^2(K_2t+C_2)}{[1+cosh^2(K_2t+C_2)^2]}$ (51)

$q = \frac{3}{cosh^2 K_2 t+C_2} - 1$ (52)

where X^2=$X_{t^2} + X_{2^2} + X_{3^2}$

From (1) and (45) we get

r= 9 $sech^2\left\{3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} t + C_2\right\}$ (53)

s= $\frac{2}{1-sech^2\left\{3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} t+C_2\right\}}$ (54)

Considering now the subleading terms in (22) at large values of A, B and C. we can get the following expressions for the energy density and pressure.

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} + \frac{C}{1+\alpha}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}} V^{-(1+\alpha)(1+A)} \qquad (55)$$

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} + \frac{C}{1+\alpha}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}} [\propto +(1+\propto)A] V^{-(1+\alpha)(1+A)} \qquad (56)$$

From (19) we have

$$\ddot{V} = 3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} V \frac{3}{2}\frac{C}{1+\propto}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}}[1-\propto +(1+\propto)A] V^{-(1+\alpha)(1+A)} \qquad (57)$$

On integration, which gives

$$\int \frac{dV}{\sqrt{K_3 V^2 K_4 V^{(1-A-\propto-\propto A)} + K_5}} = t \qquad (58)$$

Where

$K_3 \equiv 3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} K_2, K_3 \equiv \frac{3}{2}\frac{C}{1+\propto}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}}\left(\frac{1-\propto+A+\propto A}{1-A-\propto-\propto A}\right)$ and K_5 is integration constant,

Equations (55) and (56) describe the mixture of a cosmological constant equal to $\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ with matter whose equation of state is given by

$$\rho = \{\propto +(1+\propto)A\}\rho \qquad (59)$$

Which for a pure Chaplygin gas reduces to $\rho = \propto \rho$.

Now the energy, density and pressure corresponding to a scalar field $\emptyset$. The energy $\rho\emptyset$ and $\rho\emptyset$ are

$$\rho\emptyset = \frac{1}{2}\emptyset^2 + V(\emptyset) = \rho = \left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{1}{1+\alpha}} \qquad (60)$$

and

$$\rho\emptyset = \frac{1}{2}\emptyset^2 + V(\emptyset) = A\rho - \frac{B}{\rho^{\propto}}$$

$$= A\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{1}{1+\alpha}} \qquad (61)$$

From (60) and (61) we have

$$\dot{\varphi}^2 = (1+A)\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{\alpha}{1+\alpha}} \qquad (62)$$

And

$$V(\emptyset) = \frac{1}{2}(1-A)\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{1}{1+\alpha}} + \frac{B}{2}\left[\frac{B}{1+A} + \frac{C}{V^{-(1+A)(1+\propto)}}\right]^{\frac{\alpha}{1+\alpha}} \qquad (63)$$

For A=1 and $\alpha = 1$, then (58) gives

$$V = \left[\sqrt{\frac{C+K_5^2/9}{2B}} \cosh\left\{2\sqrt{3}\left(\frac{B}{2}\right)^{1/4} t\right\} - \frac{K_5}{3\sqrt{2B}}\right]^{1/2} \qquad (64)$$

Form (15)-(17) and (64), we obtain

$$A(t) = D_1[A_1 Cosh(Bt) - C]^{1/6}$$

$$\text{x exp}\left[-\frac{2X_1 i\sqrt{\frac{A_1 Cosh(A_1 i)}{A_1-A_3}} Elliptic\ F\left[\frac{iA_2 t}{2}, \frac{2A_1}{A_1-A_3}\right]}{B\sqrt{A_1 cosh(A_2 t) - A_3}}\right] \qquad (65)$$

$$B(t) = D_2[A_1 cosh\ (A_2 t) - A_3]^{1/6}$$

$$\text{x exp}\left[-\frac{2X_2 i\sqrt{\frac{A_1 Cosh(A_1 i)}{A_1-A_3}} Elliptic\ F\left[\frac{iA_2 t}{2}, \frac{2A_1}{A_1-A_3}\right]}{B\sqrt{A_1 cosh(A_2 t) - A_3}}\right] \qquad (66)$$

$$C(t) = D_3[A_1 cosh\ (A_2 t) - A_3]^{1/6}$$

$$\text{x exp}\left[-\frac{2X_3 i\sqrt{\frac{A_1 Cosh(A_1 i)}{A_1-A_3}} Elliptic\ F\left[\frac{iA_2 t}{2}, \frac{2A_1}{A_1-A_3}\right]}{B\sqrt{A_1 cosh(A_2 t) - A_3}}\right] \qquad (67)$$

where $A_1 \equiv \sqrt{\frac{C+K_5^2/9}{2B}}$, $A_2 \equiv 2\sqrt{3}\left(\frac{B}{2}\right)^{1/4}$ and $A_3 \equiv \frac{K_5}{3\sqrt{2B}}$.

The physical quantities are

$$\theta = \frac{A_1 A_2 sinh(A_2 t)}{2[A_1 cosh(A_2 t) - A_3]} \qquad (68)$$

$$Q = 6 cosech^2(A_2 t)\left[cosech\ (A_2 t) - \left(\frac{A_3}{A_1} coth\ (A_2 t)\right)\right] - 1 \qquad (69)$$

From (1) and (64), we get

$$r = 9\left(\frac{A_3^2}{A_1^2} - 1\right) cosech^2(A_2 t) + 1 \qquad (70)$$

and

$$s = \frac{\left(\frac{A_3^2}{A_1^2} - 1\right)}{\{1 - 2\cosh\ (A_2 t)\}\{A_1 \cosh\ (A_2 t)\ A_3\}} \qquad (71)$$

III- CONCLUSION

The evolution of homogeneous, anisotropic universe given by a Bianachi type-I cosmological model with modified Chaplygin gas. We have assumed that the equation of state of this modified model is valid from the radiation era to the model. We have used state-finder parameters in characterizing different phase of the model.

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